

# Symmetry in functional equations

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Here is an introduction on how to use symmetry in functional equations. We will learn by an example.

## 1 Example

Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x)f(y) = x + f(y) + 1$ .

Solution:

Note that interchanging  $x$  and  $y$ , we have

$$f(y)f(x) = y + f(x) + 1.$$

Comparing this with our original equation, we have

$$x + f(y) = y + f(x)$$

so

$$f(x) - x = f(y) - y$$

which is a constant. Hence  $f(x) = x + c$  and

$$(x + c)(y + c) = x + y + c + 1$$

which has no solutions for  $c$  so there are no solutions.

## 2 Practice

1)  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x)f(y) = f(2xy + 3) + 3f(x + y) - 3f(x) + 6(x)$ .

2)  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(xf(y) + x) = xy + f(x)$

## 3 Hints For Practice 2

Hint 1: Let  $x = 1$ .

Hint 2: Let  $x = f(x) + 1$